35th International Mathematics

TOURNAMENT OF THE TOWNS

Senior A-Level Paper **Spring 2014**

1 [**3**] Doono wrote several **1**s, placed signs "+" or "×" between every two of them, put several brackets and got 2014 in the result. His friend Dunno replaced all "+" by "×" and all "×" by "+" and also got 2014. Can this be true?

2 Is it true that any convex polygon can be dissected by a straight line into two polygons with equal perimeters and

(**a**) [**4**] equal greatest sides?

(**b**) [**4**] equal smallest sides?

3 [**6**] The King called two wizards. He ordered First Wizard to write down 100 positive real numbers (not necessarily distinct) on cards without revealing them to Second Wizard. Second Wizard must correctly determine all these numbers, otherwise both wizards will lose their heads. First Wizard is allowed to provide Second Wizard with a list of distinct numbers, each of which is either one of the numbers on the cards or a sum of some of these numbers. He is not allowed to tell which numbers are on the cards and which numbers are their sums. If Second Wizard correctly determines all 100 numbers the King tears as many hairs from each wizard's beard as the number of numbers in the list given to Second Wizard. What is the minimal number of hairs each wizard should sacrifice to stay alive?

4 [**7**] In the plane are marked all points with integer coordinates (x, y), $0 \le y \le 10$. Consider a polynomial of degree 20 with integer coefficients. Find the maximal possible number of marked points which can lie on its graph.

5 [**8**] There is a scalene triangle. Peter and Basil play the following game. On each his turn Peter chooses a point in the plane. Basil responds by painting it into red or blue. Peter wins if some triangle similar to the original one has all vertices of the same colour. Find the minimal number of moves Peter needs to win no matter how Basil would play (independently of the shape of the given triangle)?

6 [**9**] In some country every town has a unique number. In a flight directory for any two towns there is an indication whether or not they are connected by a direct non-stop flight. It is known that for any two assigned numbers M and N one can change the numeration of towns so that the town with number M gets the number N but the directory remains correct. Is it always true that for any two assigned numbers M and N one can change the numeration of towns so that the towns with numbers M and N interchange their numbers but the directory is still correct?

7 [**10**] Consider a polynomial $P(x)$ such that

$$P(0) = 1; \qquad (P(x))^2 = 1 + x + x^{100}Q(x), \text{ where } Q(x) \text{ is also a polynomial.}$$

Prove that in the polynomial $(P(x) + 1)^{100}$ the coefficient at x^{99} is zero.