International Mathematics

TOURNAMENT OF THE TOWNS

Senior O-Level Paper **Fall 2014**

1 Do there exist ten distinct positive integers such that their arithmetic mean is equal to

(a) [**1**] their greatest common divisor multiplied by 6?

(b) [**2**] their greatest common divisor multiplied by 5?

2 [**4**] The vertices of a triangle are marked by A, B, C clockwise. The triangle is rotated clockwise in a sequence: about A by $\angle A$, about B by $\angle B$, about C by $\angle C$, and so on (each time the rotation is performed about the current position of the vertex in the sequence). Prove that after six rotations the triangle returns to its initial position.

3 [**5**] From a set of 15 distinct integers Pete selects 7 numbers in all possible ways and for every selection he writes down the sum of the selected numbers. Basil, in his turn, selects 8 numbers in all possible ways and each time writes down the sum of his selected numbers. Can it happen that Pete and Basil will obtain the same set of numbers? (Each integer must be repeated in Pete's set as many times as it is repeated in Basil's set.)

4 [**5**] There are N right-angled triangles. In every given triangle Adam chose a leg and calculated the sum of the lengths of the selected legs. Then he found the total sum of the lengths of the remaining legs. Finally, he found the total sum of the hypotenuses. Given that these three numbers create a right-angled triangle, prove that all initial triangles are similar.

5 [**5**] Originally there was a pile of silver coins on a table. One can either add a gold coin and record the number of silver coins on the first list or remove a silver coin and record the number of gold coins on the second list. It happened that after several such operations only gold coins remained on the table. Prove that at that moment the sums of the numbers on the two lists were equal. We see that both sums are the same.