International Mathematics

TOURNAMENT OF THE TOWNS

Senior A-Level Paper **Fall 2014**

1 **[4]** Prove that any circumscribed polygon has three sides that can form a triangle.

2 **[6]** On a circular road there are 25 police posts equally distant. Every policeman (one at each post) has a badge with a unique number, from 1 to 25. The policemen are ordered to switch their posts so that the numbers on the badges would be in the consecutive order, from 1 to 25 clockwise. If the total sum of distances walked by the policemen along the road is minimal possible, prove that one of them remains at his initial position.

3 **[6]** Gregory wrote 100 numbers on a blackboard and calculated their product. Then he increased each number by 1 and observed that the product didn't change. He increased the numbers in the same way again, and again the product didn't change. He performed this procedure k times, each time having the same product. Find the greatest possible value of k.

4 **[7]** The circle inscribed in triangle ABC touches the sides BC, CA, AB at points A', B', C' respectively. Three lines, AA', BB' and CC' meet at point G. Define the points C_A and C_B as points of intersection of the circle circumscribed about triangle $GA'B'$ with lines AC and BC, different from B' and A'. In similar way define the points A_B, A_C, B_C, B_A. Prove that the points C_A, C_B, A_B, A_C, B_C, and B_A belong to the same circle.

5 **[7]** Pete counted all possible words consisting of m letters, such that each letter can be only one of T, O, W or N and each word contains as many T as O. Basil counted all possible words consisting of $2m$ letters such that each letter is either T or O and each word contains as many T as O. Which of the boys obtained the greater number of words?

6 **[8]** There is a wire triangle with angles x°, y°, z°. Mischievous Nick bent every side of the triangle at some point by 1 degree. In the result he got a non convex hexagon with angles $(x-1)^\circ$, 181°, $(y-1)^\circ$, 181°, $(z-1)^\circ$, 181°. Prove that the points that became the new vertices split the sides of the initial triangle in the same ratio.

7 **[10]** In one kingdom gold and platinum sands are used as currency. Exchange rate is defined by two positive integers g and p; namely, x grams of gold sand are equivalent to y grams of platinum sand if $x : y = p : g$ (x and y are not necessarily integers). At the day when the numbers were $g = p = 1001$, the Treasury announced that every following day one of the numbers, either g or p would be decreased by 1 so that after 2000 days both numbers would become equal to 1. However, the exact order in which the numbers would be decreasing was not announced. At that moment a banker had 1 kg of gold sand and 1 kg of platinum sand. The banker's goal is to perform exchanges so that by the end he would have at least 2 kg of gold sand and 2 kg of platinum sand. Can the banker fulfil his goal for certain?