International Mathematics

TOURNAMENT OF THE TOWNS

Junior O-Level Paper **Fall 2014**

1 [3] There are 99 sticks of lengths 1, 2, 3, ..., 99. Is it possible to arrange them in a shape of a rectangle?

2 Do there exist ten distinct positive integers such that their arithmetic mean is equal to

(a) [2] their greatest common divisor multiplied by 6?

(b) [2] their greatest common divisor multiplied by 5?

3 [5] Points K and L are marked on sides AB and BC of square $ABCD$ respectively so that $KB = LC$. Let P be a point of intersection of segments AL and CK. Prove that segments DP and KL are perpendicular. 1.07) L;

4 [5] During his last school year, Andrew recorded his marks in maths. He called his upcoming mark (2, 3, 4, or 5) *unexpected* if until this moment it appeared less often than any other possible mark. (For instance, if he had marks 3,4,2,5,5,5,2,3,4,3 on his list then unexpected marks would be the first 5 and the second 4). It happened that at the end of the year Andrew had on his record list forty marks and each possible mark was repeated exactly 10 times (the order of marks is unknown). Is it possible to determine the number of unexpected marks?

5 There are N right-angled triangles. In every given triangle Adam chose a leg and calculated the sum of the lengths of the selected legs. Then he found the total sum of the lengths of the remaining legs. Finally, he found the total sum of the hypotenuses. Given that these three numbers create a right-angled triangle, prove that in every given triangle the ratio of the greater leg to the smaller leg is the same. Consider the cases:

(a) [2] $N = 2$;

(b) [3] $N \geq 2$ is any positive integer.