

International Mathematics
TOURNAMENT OF THE TOWNS

Junior A-Level Paper

Fall 2014

- 1 [4] Half of entries of a square table are filled with pluses, and the remaining half are filled with minuses. Prove that either two rows, or two columns contain the same number of pluses.
- 2 [5] Prove that any circumscribed polygon has three sides that can form a triangle.
- 3 [6] Is it possible to divide all positive divisors of $100!$ (including $100!$ and 1) into two groups so that each group contains the same number of integers and the product of numbers in the first group is equal to the product of numbers in the second group?
- 4 [7] On a circular road there are 25 police posts equally distant. Every policeman (one at each post) has a badge with a unique number, from 1 to 25. The policemen are ordered to switch their posts so that the numbers on the badges would be in the consecutive order, from 1 to 25 clockwise. If the total sum of distances walked by the policemen along the road is minimal possible, prove that one of them remains at his initial position.
- 5 [8] In a right-angled triangle, two equal circles are constructed so that they touch one another and each one touches hypotenuse and one leg. Consider a segment connecting the points of tangency of the circles and the hypotenuse. Prove that the midpoint of this segment belongs to bisector of right angle of the triangle.
- 6 [8] Let us call a positive integer *plain* if it consists of the same digits (examples: 4, 111, 999999). Prove that any n -digit integer can be represented as a sum of at most $n + 1$ plain integers.
- 7 A spiderweb is a square with 100×100 nodes. 100 flies caught into the web stacked at 100 different nodes. A spider which was originally at the corner of the web crawls from a node to an adjacent node counting moves and eating flies on its way. Can the spider eat all flies in no more than
 - (a) [5] 2100 moves;
 - (b) [5] 2000 moves?