

**International Mathematics**  
**TOURNAMENT OF THE TOWNS**

**Senior O-Level Paper**

**Fall 2013**

- 1 [3] Does there exist a ten-digit number such that all its digits are different and after removing any six digits we get a composite four-digit number?
- 2 [4] On the sides of triangle  $ABC$ , three similar triangles are constructed with triangle  $YBA$  and triangle  $ZAC$  in the exterior and triangle  $XBC$  in the interior. (Above, the vertices of the triangles are ordered so that the similarities take vertices to corresponding vertices, for example, the similarity between triangle  $YBA$  and triangle  $ZAC$  takes  $Y$  to  $Z$ ,  $B$  to  $A$  and  $A$  to  $C$ ). Prove that  $AYXZ$  is a parallelogram.
- 3 [4] Denote by  $[a, b]$  the least common multiple of  $a$  and  $b$ . Let  $n$  be a positive integer such that

$$[n, n + 1] > [n, n + 2] > \cdots > [n, n + 35].$$

Prove that  $[n, n + 35] > [n, n + 36]$ .

- 4 [5] Eight rooks are placed on a chessboard so that no two rooks attack each other. Prove that one can always move all rooks, each by a move of a knight so that in the final position no two rooks attack each other as well. (In intermediate positions several rooks can share the same square).
- 5 [6] A spacecraft landed on an asteroid. It is known that the asteroid is either a ball or a cube. The rover started its route at the landing site and finished it at the point symmetric to the landing site with respect to the center of the asteroid. On its way, the rover transmitted its spatial coordinates to the spacecraft on the landing site so that the trajectory of the rover movement was known. Can it happen that this information is not sufficient to determine whether the asteroid is a ball or a cube?