International Mathematics

TOURNAMENT OF THE TOWNS

Junior O-Level Paper **Fall 2013**

1 [**3**] In a wrestling tournament, there are 100 participants, all of different strengths. The stronger wrestler always wins over the weaker opponent. Each wrestler fights twice and those who win both of their fights are given awards. What is the least possible number of awardees?

2 [**4**] Does there exist a ten-digit number such that all its digits are different and after removing any six digits we get a composite four-digit number?

3 [**4**] Denote by (a, b) the greatest common divisor of a and b. Let n be a positive integer such that

$$(n, n+1) < (n, n+2) < \cdots < (n, n+35).$$

Prove that $(n, n+35) < (n, n+36)$.

4 [**5**] Let ABC be an isosceles triangle. Suppose that points K and L are chosen on lateral sides AB and AC respectively so that $AK = CL$ and $\angle ALK + \angle LKB = 60^\circ$. Prove that $KL = BC$.

5 [**6**] Eight rooks are placed on a chessboard so that no two rooks attack each other. Prove that one can always move all rooks, each by a move of a knight so that in the final position no two rooks attack each other as well. (In intermediate positions several rooks can share the same square).