

International Mathematics
TOURNAMENT OF THE TOWNS

Junior A-Level Paper

Fall 2013

- 1 [5]** There are 100 red, 100 yellow and 100 green sticks. One can construct a triangle using any three sticks all of different colours (one red, one yellow and one green) . Prove that there is a colour such that one can construct a triangle using any three sticks of this colour.
- 2 [5]** A math teacher chose 10 consecutive numbers and submitted them to Pete and Basil. Each boy should split these numbers in pairs and calculate the sum of products of numbers in pairs. Prove that the boys can pair the numbers differently so that the resulting sums are equal.
- 3 [6]** Assume that C is a right angle of triangle ABC and N is a midpoint of the semicircle, constructed on CB as on diameter externally. Prove that AN divides the bisector of angle C in half.
- 4 [7]** There is a 8×8 table, drawn in a plane and painted in a chess board fashion. Peter mentally chooses a square and an interior point in it. Basil can draw any polygon (without self-intersections) in the plane and ask Peter whether the chosen point is inside or outside this polygon. What is the minimal number of questions sufficient to determine whether the chosen point is black or white?
- 5 [9]** A 101-gon is inscribed in a circle. From each vertex of this polygon a perpendicular is dropped to the opposite side or its extension. Prove that at least one perpendicular drops to the side.
- 6 [10]** The number
- $$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2n-1} - \frac{1}{2n}$$
- is represented as an irreducible fraction. If $3n+1$ is a prime number, prove that the numerator of this fraction is a multiple of $3n+1$.
- 7 [12]** On a table, there are 11 piles of ten stones each. Pete and Basil play the following game. In turns they take 1, 2 or 3 stones at a time: Pete takes stones from any single pile while Basil takes stones from different piles but no more than one from each. Pete moves first. The player who cannot move, loses.
- Which of the players, Pete or Basil, has a winning strategy?