

**International Mathematics
TOURNAMENT OF THE TOWNS**

Junior O-Level Paper

Spring 2012¹.

1. A treasure is buried under a square of an 8×8 board, Under each other square is a message which indicates the minimum number of steps needed to reach the square with the treasure. Each step takes one from a square to another square sharing a common side. What is the minmum number of squares we must dig up in order to bring up the treasure for sure?
2. The number 4 has an odd number of odd positive divisors, namely 1, and an even number of even positive divisors, namely 2 and 4. Is there a number with an odd number of even positive divisors and an even number of odd positive divisors?
3. In the parallelogram $ABCD$, the diagonal AC touches the incircles of triangles ABC and ADC at W and Y respectively, and the diagonal BD touches the incircles of triangles BAD and BCD at X and Z respectively. Prove that either W , X , Y and Z coincide, or $WXYZ$ is a rectangle.
4. Brackets are to be inserted into the expression $10 \div 9 \div 8 \div 7 \div 6 \div 5 \div 4 \div 3 \div 2$ so that the resulting number is an integer.
 - (a) Determine the maximum value of this integer.
 - (b) Determine the minimum value of this integer.
5. RyNo, a little rhinoceros, has 17 scratch marks on its body. Some are horizontal and the rest are vertical. Some are on the left side and the rest are on the right side. If RyNo rubs one side of its body against a tree, two scratch marks, either both horizontal or both vertical, will disappear from that side. However, at the same time, two new scratch marks, one horizontal and one vertical, will appear on the other side. If there are less than two horizontal and less than two vertical scratch marks on the side being rubbed, then nothing happens. If RyNo continues to rub its body against trees, is it possible that at some point in time, the numbers of horizontal and vertical scratch marks have interchanged on each side of its body?

Note: The problems are worth 3, 4, 4, 2+3 and 5 points respectively.

¹Courtesy of Professor Andy Liu.