International Mathematics
TOURNAMENT OF THE TOWNS

Junior A-Level Paper **Spring 2012[1].**

1. It is possible to place an even number of pears in a row such that the masses of any two neighbouring pears differ by at most 1 gram. Prove that it is then possible to put the pears two in a bag and place the bags in a row such that the masses of any two neighbouring bags differ by at most 1 gram.

2. One hundred points are marked in the plane, with no three in a line. Is it always possible to connect the points in pairs such that all fifty segments intersect one another?

3. In a team of guards, each is assigned a different positive integer. For any two guards, the ratio of the two numbers assigned to them is at least 3:1. A guard assigned the number n is on duty for n days in a row, off duty for n days in a row, back on duty for n days in a row, and so on. The guards need not start their duties on the same day. Is it possible that on any day, at least one in such a team of guards is on duty?

4. Each entry in an $n \times n$ table is either $+$ or $-$. At each step, one can choose a row or a column and reverse all signs in it. From the initial position, it is possible to obtain the table in which all signs are $+$. Prove that this can be accomplished in at most n steps.

5. Let p be a prime number. A set of $p+2$ positive integers, not necessarily distinct, is called *interesting* if the sum of any p of them is divisible by each of the other two. Determine all interesting sets.

6. A bank has one million clients, one of whom is Inspector Gadget. Each client has a unique PIN number consisting of six digits. Dr. Claw has a list of all the clients. He is able to break into the account of any client, choose any n digits of the PIN number and copy them. The n digits he copies from different clients need not be in the same n positions. He can break into the account of each client, but only once. What is the smallest value of n which allows Dr. Claw to determine the complete PIN number of Inspector Gadget?

7. Let AH be an altitude of an equilateral triangle ABC. Let I be the incentre of triangle ABH, and let L, K and J be the incentres of triangles ABI, BCI and CAI respectively. Determine $\angle KJL$.

Note: The problems are worth 4, 4, 6, 6, 8, 8 and 8 points respectively.

[1]Courtesy of Professor Andy Liu