

International Mathematics
TOURNAMENT OF THE TOWNS

Junior O-Level Paper

Spring 2011

- 1 [3] The numbers from 1 to 2010 inclusive are placed along a circle so that if we move along the circle in clockwise order, they increase and decrease alternately. Prove that the difference between some two adjacent integers is even.
- 2 [4] A rectangle is divided by 10 horizontal and 10 vertical lines into 121 rectangular cells. If 111 of them have integer perimeters, prove that they all have integer perimeters.
- 3 [5] Worms grow at the rate of 1 metre per hour. When they reach their maximal length of 1 metre, they stop growing. A full-grown worm may be dissected into two not necessarily equal parts. Each new worm grows at the rate of 1 metre per hour. Starting with 1 full-grown worm, can one obtain 10 full-grown worms in less than 1 hour?
- 4 [5] Each diagonal of a convex quadrilateral divides it into two isosceles triangles. The two diagonals of the same quadrilateral divide it into four isosceles triangles. Must this quadrilateral be a square?
- 5 A dragon gave a captured knight 100 coins. Half of them are magical, but only dragon knows which are. Each day, the knight should divide the coins into two piles (not necessarily equal in size). The day when either magic coins or usual coins are spread equally between the piles, the dragon set the knight free. Can the knight guarantee himself a freedom in at most
 - (a) [2] 50 days?
 - (b) [3] 25 days?