International Mathematics
TOURNAMENT OF THE TOWNS

Junior A-Level Paper **Spring 2011.**

1. Does there exist a hexagon that can be divided into four congruent triangles by a straight cut?

2. Passing through the origin of the coordinate plane are 180 lines, including the coordinate axes, which form 1° angles with one another at the origin. Determine the sum of the x-coordinates of the points of intersection of these lines with the line $y = -x + 100$.

3. Baron Munchausen has a set of 50 coins. The mass of each is a distinct positive integer not exceeding 100, and the total mass is even. The Baron claims that it is not possible to divide the coins into two piles with equal total mass. Can the Baron be right?

4. Given an integer $n > 1$, prove that there exist distinct positive integers a, b, c and d such that $a + b = c + d$ and $\frac{a}{b} = \frac{nc}{d}$.

5. AD and BE are altitudes of an acute triangle ABC. From D, perpendiculars are dropped to AB at G and AC at K. From E, perpendiculars are dropped to AB at F and BC at H. Prove that FG is parallel to HK and $FK = GH$.

6. Two ants crawl along the sides of the 49 squares of a 7×7 board. Each ant passes through all 64 vertices exactly once and returns to its starting point. What is the smallest possible number of sides covered by both ants?

7. In every cell of a square table is a number. The sum of the largest two numbers in each row is a and the sum of the largest two numbers in each column is b. Prove that $a = b$.

Note: The problems are worth 4, 4, 5, 6, 7, 10 and 10 points respectively.