International Mathematics

TOURNAMENT OF THE TOWNS

Junior O-Level Paper **Fall 2011.**

1. P and Q are points on the longest side AB of triangle ABC such that $AQ = AC$ and $BP = BC$. Prove that the circumcentre of triangle CPQ coincides with the incentre of triangle ABC.

2. Several guests at a round table are eating from a basket containing 2011 berries. Going in clockwise direction, each guest has eaten either twice as many berries as or six fewer berries than the next guest. Prove that not all the berries have been eaten.

3. From the 9×9 chessboard, all 16 unit squares whose row numbers and column numbers are both even have been removed. Disect the punctured board into rectangular pieces, with as few of them being unit squares as possible.

4. The vertices of a 33-gon are labelled with the integers from 1 to 33. Each edge is then labelled with the sum of the labels of its two vertices. Is it possible for the edge labels to consist of 33 consecutive numbers?

5. On a highway, a pedestrian and a cyclist were going in the same direction, while a cart and a car were coming from the opposite direction. All were travelling at different constant speeds. The cyclist caught up with the pedestrian at 10 o'clock. After a time interval, she met the cart, and after another time interval equal to the first, she met the car. After a third time interval, the car met the pedestrian, and after another time interval equal to the third, the car caught up with the cart. If the pedestrian met the car at 11 o'clock, when did he meet the cart?

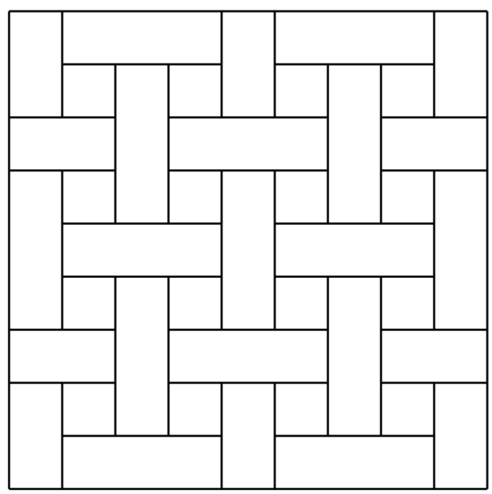
Note: The problems are worth 3, 4, 4, 4 and 5 points respectively.

Solution to Junior O-Level Fall 2011

1. The bisector of $\angle A$ is also the perpendicular bisector of CQ, and the bisector of $\angle B$ is also the perpendicular bisector of CP. The incentre of triangle ABC is the point of intersection of the bisectors of $\angle A$ and $\angle B$. The circumcentre of triangle CPQ is the point of intersection of the perpendicular bisectors of CQ and CP. Hence the incentre of triangle ABC is also the circumcentre of triangle CPQ.

2. It is not possible for each guest to eat six fewer berries than the next guest. Hence one of them has to eat twice as many, and therefore an even number of berries. Going now in the counter-clockwise direction, the next guest eats either twice as many as or six fewer than the preceding guest. It follows that every guest has eaten an even number of berries. Since 2011 is odd, not all the berries have been eaten.

3. The following diagram shows a disection of the punctured chessboard into rectangular pieces, none of them being unit squares.



4. The task is possible. Label the vertices 17, 1, 18, 2, 19, 3, ..., 15, 32, 16 and 33. Then the edge labels are 18, 19, 20, 21, 22, ..., 47, 48, 49 and 50.

5. The diagram below shows five snapshots of the highway. Since all speeds are constant, the motions can be represented by straight lines, AD for the pedestrian, AC for the cyclist, BE for the cart and CE for the car. The equality of time intervals yield $AB = BC$ and $CD = DE$. Hence F, which represents the moment the pedestrian met the cart, is the centroid of triangle ACE, so that $AF = \frac{2}{3}AD$. Since A is at 10 o'clock and D is at 11 o'clock, F is at 10:40.