INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Junior A-Level Paper, Fall 2011.

Grades 8 – 10

(The result is computed from the three problems with the highest scores.)

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points	problems
3	1. An integer $N > 1$ is written on the board. Alex writes a sequence of positive integers, obtaining new integers in the following manner: he takes any divisor greater than 1 of the last number and either adds it to, or subtracts it from the number itself. Is it always (for all $N > 1$) possible for Alex to write the number 2011 at some point?
4	2. On side AB of triangle ABC a point P is taken such that $AP = 2PB$. It is known that $CP = 2PQ$ where Q is the midpoint of AC. Prove that ABC is a right triangle.
5	3. A balance and a set of pairwise different weights are given. It is known that for any pair of weights from this set put on the left pan of the balance, one can counterbalance them by one or several of the remaining weights put on the right pan. Find the least possible number of weights in the set.
6	4. A checkered table consists of 2012 rows and $k > 2$ columns. A marker is placed in a cell of the left-most column. Two players move the marker in turns. During each move, the player moves the marker by 1 cell to the right, up or down to a cell that had never been occupied by the marker before. The game is over when any of the players moves the marker to the right-most column. However, whether this player is to win or to lose, the players are advised only when the marker reaches the second column from the right. Can any player secure his win?
6	5. Given that $0 < a, b, c, d < 1$ and $abcd = (1-a)(1-b)(1-c)(1-d)$, prove that $$(a+b+c+d) - (a+c)(b+d) \geq 1.$$
7	6. A car goes along a straight highway at the speed of 60 km per hour. A 100 meter long fence is standing parallel to the highway. Every second, the passenger of the car measures the angle of vision of the fence. Prove that the sum of all angles measured by him is less than 1100 degrees.
9	7. The vertices of a regular 45-gon are painted into three colors so that the number of vertices of each color is the same. Prove that three vertices of each color can be selected so that three triangles formed by the chosen vertices of the same color are all equal.