

International Mathematics
TOURNAMENT OF THE TOWNS

Junior O-Level Paper

Spring 2010

- 1 [3] Each of six fruit baskets contains pears, plums and apples. The number of plums in each basket equals the total number of apples in all other baskets combined while the number of apples in each basket equals the total number of pears in all other baskets combined. Prove that the total number of fruits is a multiple of 31.

- 2 [3] Karlson and Smidge divide a cake in a shape of a square in the following way. First, Karlson places a candle on the cake (chooses some interior point). Then Smidge makes a straight cut from the candle to the boundary in the direction of his choice. Then Karlson makes a straight cut from the candle to the boundary in the direction perpendicular to Smidge's cut. As a result, the cake is split into two pieces; Smidge gets the smaller one. Smidge wants to get a piece which is no less than a quarter of the cake. Can Karlson prevent Smidge from getting the piece of that size?

- 3 An angle is given in a plane. Using only a compass, one must find out
 - (a) [2] if this angle is acute. Find the minimal number of circles one must draw to be sure.
 - (b) [2] if this angle equals 31° . (One may draw as many circles as one needs.)

- 4 [5] At the math contest each participant met at least 3 pals who he/she already knew. Prove that the Jury can choose an even number of participants (more than two) and arrange them around a table so that each participant be set between these who he/she knows.

- 5 [5] 101 numbers are written on a blackboard: $1^2, 2^2, \dots, 101^2$. Alex chooses any two numbers and replaces them by their positive difference. He repeats this operation until one number is left on the blackboard. Determine the smallest possible value of this number.