International Mathematics

TOURNAMENT OF THE TOWNS

Junior A-Level Paper **Spring 2010**

1 [**3**] Alex has a piece of cheese. He chooses a positive number $a \neq 1$ and cuts the piece into several pieces one by one. Every time he choses a piece and cuts it in the same ratio $1 : a$. His goal is to divide the cheese into two piles of equal masses. Can he do it?

2 [**4**] Let M be the midpoint of side AC of the triangle ABC. Let P be a point on the side BC. If O is the point of intersection of AP and BM and $BO = BP$, determine the ratio OM/PC .

3 Each of 999 numbers placed in a circular way is either 1 or -1. (Both values appear). Consider the total sum of the products of every 10 consecutive numbers.

(a) [**3**] Find the minimal possible value of this sum.

(b) [**3**] Find the maximal possible value of this sum.

4 [**6**] Can it happen that the sum of digits of some positive integer n equals 100 while the sum of digits of number n^3 equals 100^3?

5 N horsemen are riding in the same direction along a circular road. Their speeds are constant and pairwise distinct. There is a single point on the road where the horsemen can surpass one another. Can they ride in this fashion for arbitrarily long time ? Consider the cases:

(a) [**3**] $N = 3$;

(b) [**5**] $N = 10$.

6 [**8**] A broken line consists of 31 segments. It has no self intersections, and its start and end points are distinct. All segments are extended to become straight lines. Find the least possible number of straight lines.

7 [**11**] Several fleas sit on the squares of a 10×10 chessboard (at most one flea per square). Every minute, all fleas simultaneously jump to adjacent squares. Each flea begins jumping in one of four directions (up, down, left, right), and keeps jumping in this direction while it is possible; otherwise, it reverses direction on the opposite. It happened that during one hour no two fleas ever occupied the same square. Find the maximal possible number of fleas on the board.