

International Mathematics
TOURNAMENT OF THE TOWNS

Junior O-Level Paper

Fall 2008

- 1 [3] Each of ten boxes contains a different number of pencils. No two pencils in the same box are of the same colour. Prove that one can choose one pencil from each box so that no two are of the same colour.
- 2 [3] Twenty-five of the numbers $1, 2, \dots, 50$ are chosen. Twenty-five of the numbers $51, 52, \dots, 100$ are also chosen. No two chosen numbers differ by 0 or 50. Find the sum of all 50 chosen numbers.
- 3 [4] Acute triangle $A_1A_2A_3$ is inscribed in a circle of radius 2. Prove that one can choose points B_1, B_2, B_3 on the arcs A_1A_2, A_2A_3, A_3A_1 respectively, such that the numerical value of the area of the hexagon $A_1B_1A_2B_2A_3B_3$ is equal to the numerical value of the perimeter of the triangle $A_1A_2A_3$.
- 4 [4] Given three distinct positive integers such that one of them is the average of the two others. Can the product of these three integers be the perfect 2008th power of a positive integer?
- 5 [4] On a straight track are several runners, each running at a different constant speed. They start at one end of the track at the same time. When a runner reaches any end of the track, he immediately turns around and runs back with the same speed (then he reaches the other end and turns back again, and so on). Some time after the start, all runners meet at the same point. Prove that this will happen again.