1 [4] 100 Queens are placed on a $100 \times 100$ chessboard so that no two attack each other. Prove that each of four $50 \times 50$ corners of the board contains at least one Queen.

2 [6] Each of 4 stones weights the integer number of grams. A balance with arrow indicates the difference of weights on the left and the right sides of it. Is it possible to determine the weights of all stones in 4 weighings, if the balance can make a mistake in 1 gram in at most one weighing?

3 [6] In his triangle $ABC$ Serge made some measurements and informed Ilias about the lengths of median $AD$ and side $AC$. Based on these data Ilias proved the assertion: angle $CAB$ is obtuse, while angle $DAB$ is acute. Determine a ratio $AD/AC$ and prove Ilias’ assertion (for any triangle with such a ratio).

4 [6] Baron Münchausen claims that he got a map of a country that consists of five cities. Each two cities are connected by a direct road. Each road intersects no more than one another road (and no more than once). On the map, the roads are colored in yellow or red, and while circling any city (along its border) one can notice that the colors of crossed roads alternate. Can Baron’s claim be true?

5 [8] Let $a_1, \ldots, a_n$ be a sequence of positive numbers, so that $a_1 + a_2 + \cdots + a_n \leq 1/2$. Prove that $(1 + a_1)(1 + a_2)\ldots(1 + a_n) < 2$.

6 [9] Let $ABC$ be a non-isosceles triangle. Two isosceles triangles $AB'C$ with base $AC$ and $CA'B$ with base $BC$ are constructed outside of triangle $ABC$. Both triangles have the same base angle $\varphi$. Let $C_1$ be a point of intersection of the perpendicular from $C$ to $A'B'$ and the perpendicular bisector of the segment $AB$. Determine the value of $\angle AC_1B$.

7 In an infinite sequence $a_1, a_2, a_3, \ldots$, the number $a_1$ equals 1, and each $a_n$, $n > 1$, is obtained from $a_{n-1}$ as follows:

- if the greatest odd divisor of $n$ has residue 1 modulo 4, then $a_n = a_{n-1} + 1$,
- and if this residue equals 3, then $a_n = a_{n-1} - 1$.

Prove that in this sequence

(a) [5] the number 1 occurs infinitely many times;

(b) [5] each positive integer occurs infinitely many times.

(The initial terms of this sequence are 1, 2, 1, 2, 3, 2, 1, 2, 3, 4, 3, \ldots)