

**International Mathematics  
TOURNAMENT OF THE TOWNS**

**Junior O-Level Paper<sup>1</sup>**

**Spring 2007.**

1. The sides of a convex pentagon are extended on both sides to form five triangles. If these triangles are congruent to one another, does it follow that the pentagon is regular?
2. Two 2007-digit numbers are given. It is possible to delete 7 digits from each of them to obtain the same 2000-digit number. Prove that it is also possible to insert 7 digits into the given numbers so as to obtain the same 2014-digit number.
3. What is the least number of rooks that can be placed on a standard  $8 \times 8$  chessboard so that all the white squares are attacked? (A rook also attacks the square it is on, in addition to every other square in the same row or column.)
4. Three nonzero real numbers are given. If they are written in any order as coefficients of a quadratic trinomial, then each of these trinomials has a real root. Does it follow that each of these trinomials has a positive root?
5. A triangular pie has the same shape as its box, except that they are mirror images of each other. We wish to cut the pie in two pieces which can fit together in the box without turning either piece over. How can this be done if
  - (a) one angle of the triangle is three times as big as another;
  - (b) one angle of the triangle is obtuse and is twice as big as one of the acute angles?

**Note:** The problems are worth 4, 4, 4, 4 and 1+4 points respectively.

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<sup>1</sup>Courtesy of Professor Andy Liu