

**International Mathematics
TOURNAMENT OF THE TOWNS**

Junior A-Level Paper¹

Spring 2007.

1. Let n be a positive integer. In order to find the integer closest to \sqrt{n} , Mary finds a^2 , the closest perfect square to n . She thinks that a is then the number she is looking for. Is she always correct?
2. K , L , M and N are points on sides AB , BC , CD and DA , respectively, of the unit square $ABCD$ such that KM is parallel to BC and LN is parallel to AB . The perimeter of triangle KLB is equal to 1. What is the area of triangle MND ?
3. Anna's number is obtained by writing down 20 consecutive positive integers, one after another in arbitrary order. Bob's number is obtained in the same way, but with 21 consecutive positive integers. Can they obtain the same number?
4. Several diagonals (possibly intersecting each other) are drawn in a convex n -gon in such a way that no three diagonals intersect in one point. If the n -gon is cut into triangles, what is the maximum possible number of these triangles?
5. Find all (finite) increasing arithmetic progressions, consisting only of prime numbers, such that the number of terms is larger than the common difference.
6. In the quadrilateral $ABCD$, $AB = BC = CD$ and $\angle BMC = 90^\circ$, where M is the midpoint of AD . Determine the acute angle between the lines AC and BD .
7. Nancy shuffles a deck of 52 cards and spreads the cards out in a circle face up, leaving one spot empty. Andy, who is in another room and does not see the cards, names a card. If this card is adjacent to the empty spot, Nancy moves the card to the empty spot, without telling Andy; otherwise nothing happens. Then Andy names another card and so on, as many times as he likes, until he says "stop."
 - (a) Can Andy guarantee that after he says "stop," no card is in its initial spot?
 - (b) Can Andy guarantee that after he says "stop," the Queen of Spades is not adjacent to the empty spot?

Note: The problems are worth 3, 4, 5, 6, 7, 8 and 5+5 points respectively.

¹Courtesy of Professor Andy Liu