

**International Mathematics
TOURNAMENT OF THE TOWNS**

Senior O-Level Paper

Fall 2007²

- 1 [3] Pictures are taken of 100 adults and 100 children, with one adult and one child in each, the adult being the taller of the two. Each picture is reduced to $\frac{1}{k}$ of its original size, where k is a positive integer which may vary from picture to picture. Prove that it is possible to have the reduced image of each adult taller than the reduced image of every child.
- 2 Initially, the number 1 and two positive numbers x and y are written on a blackboard. In each step, we can choose two numbers on the blackboard, not necessarily different, and write their sum or their difference on the blackboard. We can also choose a non-zero number of the blackboard and write its reciprocal on the blackboard. Is it possible to write on the blackboard, in a finite number of moves, the number
 - a) [2] x^2 ;
 - b) [2] xy ?
- 3 [4] Give a construction by straight-edge and compass of a point C on a line ℓ parallel to a segment AB , such that the product $AC \cdot BC$ is minimum.
- 4 [4] The audience chooses two of twenty-nine cards, numbered from 1 to 29 respectively. The assistant of a magician chooses two of the remaining twenty-seven cards, and asks a member of the audience to take them to the magician, who is in another room. The two cards are presented to the magician in an arbitrary order. By an arrangement with the assistant beforehand, the magician is able to deduce which two cards the audience has chosen only from the two cards he receives. Explain how this may be done.
- 5 A square of side length 1 centimetre is cut into three convex polygons. Is it possible that the diameter of each of them does not exceed
 - a) [1] 1 centimetre;
 - b) [2] 1.01 centimetres;
 - c) [2] 1.001 centimetres?

²Courtesy of Professor Andy Liu.