

International Mathematics
TOURNAMENT OF THE TOWNS

A-Level Paper

Spring 2006.²

- 1 [4] Prove that one can always mark 50 points inside of any convex 100-gon, so that each its vertex is on a straight line connecting some two marked points.
- 2 [5] Are there exist some positive integers n and k , such that the first decimals of 2^n (from left to the right) represent the number 5^k while the first decimals of 5^n represent the number 2^k ?
- 3 [5] Consider a polynomial $P(x) = x^4 + x^3 - 3x^2 + x + 2$. Prove that at least one of the coefficients of $(P(x))^k$, (k is any positive integer) is negative.
- 4 [6] In triangle ABC let X be some fixed point on bisector AA' while point B' be intersection of BX and AC and point C' be intersection of CX and AB . Let point P be intersection of segments $A'B'$ and CC' while point Q be intersection of segments $A'C'$ and BB' . Prove that $\angle PAC = \angle QAB$.
- 5 [6] Prove that one can find infinite number of distinct pairs of integers such that every digit of each number is no less than 7 and the product of two numbers in each pair is also a number with all its digits being no less than 7.
- 7 On a circumference at some points sit 12 grasshoppers. The points divide the circumference into 12 arcs. By a signal each grasshopper jumps from its point to the midpoint of its arc (in clockwise direction). In such way new arcs are created. The process repeats for a number of times. Can it happen that at least one of the grasshoppers returns to its initial point after
 - a) [4] 12 jumps?
 - a) [3] 13 jumps?
- 8 [8] An ant crawls along a closed route along the edges of a dodecahedron, never going backwards. Each edge of the route is passed exactly twice. Prove that one of the edges is passed both times in the same direction. (Dodecahedron has 12 faces in the shape of pentagon, 30 edges and 20 vertices; each vertex emitting 3 edges).

²Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [].