Juniors

(Grades up to 10)

International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper **Spring 2006.**[1]

1 [**3**] Let $\angle A$ in a triangle ABC be $60°$. Let point N be the intersection of AC and perpendicular bisector to the side AB while point M be the intersection of AB and perpendicular bisector to the side AC. Prove that $CB = MN$.

2 [**3**] A $n \times n$ table is filled with the numbers as follows: the first column is filled with 1's, the second column with 2's, and so on. Then, the numbers on the main diagonal (from top-left to bottom-right) are erased. Prove that the total sums of the numbers on both sides of the main diagonal differ in exactly two times.

3 [**4**] Let a be some positive number. Find the number of integer solutions x of inequality $2 < xa < 3$ given that inequality $1 < xa < 2$ has exactly 3 integer solutions. Consider all possible cases.

4 Anna, Ben and Chris sit at the round table passing and eating nuts. At first only Anna has the nuts that she divides equally between Ben and Chris, eating a leftover (if there is any). Then Ben does the same with his pile. Then Chris does the same with his pile. The process repeats itself: each of the children divides his/her pile of nuts equally between his/her neighbours eating the leftovers if there are any. Initially, the number of nuts is large enough (more than 3). Prove that

a) [**3**] at least one nut is eaten;

b) [**3**] all nuts cannot be eaten.

5 Pete has n^3 white cubes of the size $1 \times 1 \times 1$. He wants to construct a $n \times n \times n$ cube with all its faces being completely white. Find the minimal number of the faces of small cubes that Basil must paint (in black colour) in order to prevent Pete from fulfilling his task. Consider the cases:

a) [**2**] $n = 2$;

b) [**4**] $n = 3$.

[1]Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [ ].