Juniors

(Grades up to 10)

International Mathematics
TOURNAMENT OF THE TOWNS

A-Level Paper **Fall 2006.**[1]

1 [3] Two regular polygons, a 7-gon and a 17-gon are given. For each of them two circles are drawn, an inscribed circle and a circumscribed circle. It happened that rings containing the polygons have equal areas. Prove that sides of the polygons are equal.

2 [5] When Ann meets new people, she tries to find out who is acquainted with who. In order to memorize it she draws a circle in which each person is depicted by a chord; moreover, chords corresponding to acquainted persons intersect (possibly at the ends), while the chords corresponding to non-acquainted persons do not. Ann believes that such set of chords exists for any company. Is her judgement correct?

3 A 3×3 square is filled with numbers: a, b, c, d, e, f, g, h, i in the following way: Given that the square is magic (sums of the numbers in each row, column and each of two diagonals are the same), show that

a	b	c
d	e	f
g	h	i

a) [3] $2(a+c+g+i) = b+d+f+h+4e$.

b) [3] $2(a^3+c^3+g^3+i^3) = b^3+d^3+f^3+h^3+4e^3$.

4 [6] A circle of radius R is inscribed into an acute triangle. Three tangents to the circle split the triangle into three right angle triangles and a hexagon that has perimeter Q. Find the sum of diameters of circles inscribed into the three right triangles.

5 Consider a square painting of size 1×1. A rectangular sheet of paper of area 2 is called its "envelope" if one can wrap the painting with it without cutting the paper. (For instance, a 2×1 rectangle and a square with side $\sqrt{2}$ are envelopes.)

a) [4] Show that there exist other envelopes.

b) [3] Show that there exist infinitely many envelopes.

6 [8] Let $1 + 1/2 + 1/3 + \cdots + 1/n = a_n/b_n$, where a_n and b_n are relatively prime. Show that there exist infinitely many positive integers n, such that $b_{n+1} < b_n$.

7 [9] A Magician has a deck of 52 cards. Spectators want to know the order of cards in the deck (without specifying face-up or face-down). They are allowed to ask the questions "How many cards are there between such-and-such card and such-and-such card?" One of the spectators knows the card order. Find the minimal number of questions he needs to ask to be sure that the other spectators can learn the card order.

---

[1] Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [ ].