

**International Mathematics
TOURNAMENT OF THE TOWNS**

Senior O-Level Paper¹

Spring 2005.

1. The graphs of four functions of the form $y = x^2 + ax + b$, where a and b are real coefficients, are plotted on the coordinate plane. These graphs have exactly four points of intersection, and at each one of them, exactly two graphs intersect. Prove that the sum of the largest and the smallest x -coordinates of the points of intersection is equal to the sum of the other two.
2. The base-ten expressions of all the positive integers are written on an infinite ribbon without spacing: 1234567891011... Then the ribbon is cut up into strips seven digits long. Prove that any seven digit integer will:
 - (a) appear on at least one of the strips;
 - (b) appear on an infinite number of strips.
3. M and N are the midpoints of sides BC and AD , respectively, of a square $ABCD$. K is an arbitrary point on the extension of the diagonal AC beyond A . The segment KM intersects the side AB at some point L . Prove that $\angle KNA = \angle LNA$.
4. In a certain big city, all the streets go in one of two perpendicular directions. During a drive in the city, a car does not pass through any place twice, and returns to the parking place along a street from which it started. If it has made 100 left turns, how many right turns must it have made?
5. The sum of several positive numbers is equal to 10, and the sum of their squares is greater than 20. Prove that the sum of the cubes of these numbers is greater than 40.

Note: The problems are worth 3, 3+1, 4, 4 and 5 points respectively.

¹Courtesy of Andy Liu.