International Mathematics
TOURNAMENT OF THE TOWNS

Junior O-Level Paper[1] **Spring 2005.**

1. Anna and Boris move simultaneously towards each other, from points A and B respectively. Their speeds are constant, but not necessarily equal. Had Anna started 30 minutes earlier, they would have met 2 kilometers nearer to B. Had Boris started 30 minutes earlier instead, they would have met some distance nearer to A. Can this distance be uniquely determined?

2. Prove that one of the digits 1, 2 and 9 must appear in the base-ten expression of n or $3n$ for any positive integer n.

3. There are eight identical Black Queens in the first row of a chessboard and eight identical White Queens in the last row. The Queens move one at a time, horizontally, vertically or diagonally by any number of squares as long as no other Queens are in the way. Black and White Queens move alternately. What is the minimal number of moves required for interchanging the Black and White Queens?

4. M and N are the midpoints of sides BC and AD, respectively, of a square $ABCD$. K is an arbitrary point on the extension of the diagonal AC beyond A. The segment KM intersects the side AB at some point L. Prove that $\angle KNA = \angle LNA$.

5. In a certain big city, all the streets go in one of two perpendicular directions. During a drive in the city, a car does not pass through any place twice, and returns to the parking place along a street from which it started. If it has made 100 left turns, how many right turns must it have made?

Note: The problems are worth 3, 4, 5, 5 and 5 points respectively.

[1] Courtesy of Andy Liu.