International Mathematics
TOURNAMENT OF THE TOWNS

Junior A-Level Paper[1] **Spring 2005.**

1. On the graph of a polynomial with integral coefficients are two points with integral coordinates. Prove that if the distance between these two points is integral, then the segment connecting them is parallel to the x-axis.

2. The altitudes AD and BE of triangle ABC meet at its orthocentre H. The midpoints of AB and CH are X and Y, respectively. Prove that XY is perpendicular to DE.

3. Baron Münchhausen's watch works properly, but has no markings on its face. The hour, minute and second hands have distinct lengths, and they move uniformly. The Baron claims that since none of the mutual positions of the hands is repeats twice in the period between 8:00 and 19:59, he can use his watch to tell the time during the day. Is his assertion true?

4. A 10×12 paper rectangle is folded along the grid lines several times, forming a thick 1×1 square. How many pieces of paper can one possibly get by cutting this square along the segment connecting

   (a) the midpoints of a pair of opposite sides;

   (b) the midpoints of a pair of adjacent sides?

5. In a rectangular box are a number of rectangular blocks, not necessarily identical to one another. Each block has one of its dimensions reduced. Is it always possible to pack these blocks in a smaller rectangular box, with the sides of the blocks parallel to the sides of the box?

6. John and James wish to divide 25 coins, of denominations 1, 2, 3, ..., 25 kopeks. In each move, one of them chooses a coin, and the other player decides who must take this coin. John makes the initial choice of a coin, and in subsequent moves, the choice is made by the player having more kopeks at the time. In the event that there is a tie, the choice is made by the same player in the preceding move. After all the coins have been taken, the player with more kokeps wins. Which player has a winning strategy?

7. The squares of a chessboard are numbered in the following way. The upper left corner is numbered 1. The two squares on the next diagonal from top-right to bottom-left are numbered 2 and 3. The three squares on the next diagonal are numbered 4, 5 and 6, and so on. The two squares on the second-to-last diagonal are numbered 62 and 63, and the lower right corner is numbered 64. Peter puts eight pebbles on the squares of the chessboard in such a way that there is exactly one pebble in each column and each row. Then he moves each pebble to a square with a number greater than that of the original square. Can it happen that there is still exactly one pebble in each column and each row?

Note: The problems are worth 4, 5, 5, 2+4, 6, 6 and 8 points respectively.

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[1] Courtesy of Andy Liu.