Seniors
(Grades 11, 12)

International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper **Fall 2005.**[1]

1 [3] Can two perfect cubes fit between two consecutive perfect squares? In other words, do there exist positive integers a, b, n such that $n^2 < a^3 < b^3 < (n+1)^2$?

2 [3] A segment of length $\sqrt{2}+\sqrt{3}+\sqrt{5}$ is drawn. Is it possible to draw a segment of unit length using a compass and a straightedge?

3 [4] Among 6 coins one is counterfeit (its weight differs from that real one and neither weights is known). Using scales that show the total weight of coins placed on the cup, find the counterfeit coin in 3 weighings.

4 [4] On all three sides of a right triangle ABC external squares are constructed; their centers denoted by D, E, F. Show that the ratio of the area of triangle DEF to the area of triangle ABC is:

a) [2] greater than 1;

b) [2] at least 2.

5 [5] A cube lies on the plane. After being rolled a few times (over its edges), it is brought back to its initial location with the same face up. Could the top face have been rotated by 90 degrees?

[1]Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [ ].