Seniors

(Grades 11, 12)

International Mathematics
TOURNAMENT OF THE TOWNS

A-Level Paper **Fall 2005.**[1]

1 [**3**] For which $n \geq 2$ can one find a sequence of distinct positive integers $a_1, a_2, \ldots, a_n$ so that the sum

$$\frac{a_1}{a_2} + \frac{a_2}{a_3} + ... + \frac{a_n}{a_1}$$

is an integer?

2 [**5**] Two ants crawl along the perimeter of a polygonal table, so that the distance between them is always 10 cm. Each side of the table is more than 1 meter long. At the initial moment both ants are on the same side of the table.

(a) [**2**] Suppose that the table is a convex polygon. Is it always true that both ants can visit each point on the perimeter?

(b) [**3**] Is it always true (this time without assumption of convexity) that each point on the perimeter can be visited by at least one ant?

3 [**5**] Originally, every square of 8×8 chessboard contains a rook. One by one, rooks which attack an odd number of others are removed. Find the maximal number of rooks that can be removed. (A rook attacks another rook if they are on the same row or column and there are no other rooks between them.)

4 [**6**] Several positive numbers each not exceeding 1 are written on the circle. Prove that one can divide the circle into three arcs so that the sums of numbers on any two arcs differ by no more than 1. (If there are no numbers on an arc, the sum is equal to zero.)

5 [**7**] In triangle ABC bisectors AA_1, BB_1 and CC_1 are drawn. Given $\angle A : \angle B : \angle C = 4 : 2 : 1$, prove that $A_1B_1 = A_1C_1$.

6 [**8**] Two operations are allowed:

(i) to write two copies of number 1;

(ii) to replace any two identical numbers n by $(n + 1)$ and $(n - 1)$.

Find the minimal number of operations that required to produce the number 2005 (at the beginning there are no numbers).

[1] Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [ ].