

**International Mathematics
TOURNAMENT OF THE TOWNS**

Senior A-Level Paper

Spring 2004.

1. Each day, the price of the shares of the corporation “Soap Bubble, Limited” either increases or decreases by n percent, where n is an integer such that $0 < n < 100$. The price is calculated with unlimited precision. Does there exist an n for which the price can take the same value twice?
2. All angles of a polygonal billiard table have measures in integral numbers of degrees. A tiny billiard ball rolls out of the vertex A of an interior 1° angle and moves inside the billiard table, bouncing off its sides according to the law “angle of reflection equals angle of incidence”. If the ball passes through a vertex, it will drop in and stays there. Prove that the ball will never return to A .
3. The perpendicular projection of a triangular pyramid on some plane has the largest possible area. Prove that this plane is parallel to either a face or two opposite edges of the pyramid.
4. At the beginning of a two-player game, the number $2004!$ is written on the blackboard. The players move alternately. In each move, a positive integer smaller than the number on the blackboard and divisible by at most 20 different prime numbers is chosen. This is subtracted from the number on the blackboard, which is erased and replaced by the difference. The winner is the player who obtains 0. Does the player who goes first or the one who goes second have a guaranteed win, and how should that be achieved?
5. The parabola $y = x^2$ intersects a circle at exactly two points A and B . If their tangents at A coincide, must their tangents at B also coincide?
6. The audience shuffles a deck of 36 cards, containing 9 cards in each of the suits spades, hearts, diamonds and clubs. A magician predicts the suit of the cards, one at a time, starting with the uppermost one in the face-down deck. The design on the back of each card is an arrow. An assistant examines the deck without changing the order of the cards, and points the arrow on the back each card either towards or away from the magician, according to some system agreed upon in advance with the magician. Is there such a system which enables the magician to guarantee the correct prediction of the suit of at least
 - (a) 19 cards;
 - (b) 20 cards?

Note: The problems are worth 4, 6, 6, 6, 7 and 3+5points respectively.