Juniors
(Grades up to 10)

International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper **Spring 2004.**

1 **[3]** In triangle ABC the bisector of angle A, the perpendicular to side AB from its midpoint, and the altitude from vertex B, intersect in the same point. Prove that the bisector of angle A, the perpendicular to side AC from its midpoint, and the altitude from vertex C also intersect in the same point.

2 **[3]** Find all possible values of $n \geq 1$ for which there exist n consecutive positive integers whose sum is a prime number.

3 Bucket A contains 3 litres of syrup. Bucket B contains n litres of water. Bucket C is empty. We can perform any combination of the following operations:

- Pour away the entire amount in bucket X,
- Pour the entire amount in bucket X into bucket Y,
- Pour from bucket X into bucket Y until buckets Y and Z contain the same amount.

(a) **[3]** How can we obtain 10 litres of 30% syrup if $n = 20$?

(b) **[2]** Determine all possible values of n for which the task in (a) is possible.

4 **[5]** A positive integer $a > 1$ is given (in decimal notation). We copy it twice and obtain a number $b = \overline{aa}$ which happened to be a multiple of a^2. Find all possible values of b/a^2.

5 **[6]** Two 10-digit integers are called neighbours if they differ in exactly one digit (for example, integers 1234567890 and 1234507890 are neighbours). Find the maximal number of elements in the set of 10-digit integers with no two integers being neighbours.