

**International Mathematics  
TOURNAMENT OF THE TOWNS**

**Junior A-Level Paper**

**Spring 2004.**

1. The sum of all terms of a finite arithmetical progression of integers is a power of two. Prove that the number of terms is also a power of two.
2. What is the maximal number of checkers that can be placed on an  $8 \times 8$  checkerboard so that each checker stands on the middle one of three squares in a row diagonally, with exactly one of the other two squares occupied by another checker?
3. Each day, the price of the shares of the corporation “Soap Bubble, Limited” either increases or decreases by  $n$  percent, where  $n$  is an integer such that  $0 < n < 100$ . The price is calculated with unlimited precision. Does there exist an  $n$  for which the price can take the same value twice?
4. Two circles intersect in points  $A$  and  $B$ . Their common tangent nearer  $B$  touches the circles at points  $E$  and  $F$ , and intersects the extension of  $AB$  at the point  $M$ . The point  $K$  is chosen on the extension of  $AM$  so that  $KM = MA$ . The line  $KE$  intersects the circle containing  $E$  again at the point  $C$ . The line  $KF$  intersects the circle containing  $F$  again at the point  $D$ . Prove that the points  $A$ ,  $C$  and  $D$  are collinear.
5. All sides of a polygonal billiard table are in one of two perpendicular directions. A tiny billiard ball rolls out of the vertex  $A$  of an inner  $90^\circ$  angle and moves inside the billiard table, bouncing off its sides according to the law “angle of reflection equals angle of incidence”. If the ball passes a vertex, it will drop in and stays there. Prove that the ball will never return to  $A$ .
6. At the beginning of a two-player game, the number  $2004!$  is written on the blackboard. The players move alternately. In each move, a positive integer smaller than the number on the blackboard and divisible by at most 20 different prime numbers is chosen. This is subtracted from the number on the blackboard, which is erased and replaced by the difference. The winner is the player who obtains 0. Does the player who goes first or the one who goes second have a guaranteed win, and how should that be achieved?

**Note:** The problems are worth 4, 5, 5, 6, 6 and 7 points respectively.