Seniors
(Grades 11 & 12)

International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper **Fall 2004.**[1]

1 [**3**] Three circles pass through point X and A, B, C are their intersection points (other than X). Let A' be the second point of intersection of straight line AX and the circle circumscribed around triangle BCX. Define simiarly points B', C'. Prove that triangles ABC', $AB'C$, and $A'BC$ are similar.

2 [**3**] A box contains red, blue, and white balls; 100 balls in total. It is known that among any 26 of them there are always 10 balls of the same color.

Find the minimal number N such that among any N balls there are always 30 balls of the same color.

3 [**4**] $P(x)$ and $Q(x)$ are polynomials of positive degree such that

$$P(P(x)) = Q(Q(x)) \qquad \text{and} \qquad P(P(P(x))) = Q(Q(Q(x))) \qquad \text{for all } x.$$

Does this necessarily mean that $P(x) = Q(x)$?

4 [**4**] Find the number of ways to decompose 2004 into a sum of positive integers (one or more) that all are "approximately equal".

Decompostions obtained from one another by permutations are not considered as different.

Two numbers are called *approximately equal* if their difference is at most 1.

5 [**5**] Find all values N such that it is possible to arrange all integers from 1 to N in a way that for any group of two or more consecutive numbers the arithmetic mean of this group is not an integer.

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[1]Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [ ].