

International Mathematics
TOURNAMENT OF THE TOWNS

A-Level Paper

Fall 2004.¹

- 1 [4] A triangle is called *rational* if all its angles (measured in grades) are rational. An interior point of the triangle is called *rational* if all three triangles obtained by connecting this point with three vertices are rational.

Prove that every acute rational triangle has at least three distinct rational points.

- 2 [5] An incircle of triangle ABC touches the sides BC , CA and AB at points A' , B' and C' respectively.

Is it necessarily true that triangle ABC is equilateral if $AA' = BB' = CC'$?

- 3 [6] What is the maximal number of knights one can place on 8×8 chessboard so that each knight attacks no more than 7 other knights?

- 4 [6] The results of operations

$$x + y, \quad x - y, \quad xy, \quad x/y$$

are written on four cards that are placed on a table in random order. Prove that one can restore both x and y given that x and y are positive numbers.

- 5 [7] Point K belongs to side BC of triangle ABC . Incircles of triangles ABK and ACK touch BC at points M and N respectively. Prove that $BM \cdot CN > KM \cdot KN$.

- 6 [8] Joe and Pete, in turns, divide a piece of cheese. At first, Joe cuts the cheese into two pieces, then Pete chooses one of them and cuts it into two pieces. Then Joe chooses and cuts. The procedure continues until they get 5 pieces.

Now starting from Joe they, in turns, pick up pieces until nothing is left. What is the maximal amount of cheese that each of them can guarantee for himself (no matter how his opponent plays)?

- 7 [8] There are two rectangles A and B . It is known that one can tile a rectangle similar to B using copies of A . Prove that one can tile a rectangle similar to A using copies of B .

¹Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [].