

Seniors

(Grades 11 and up)

International Mathematics TOURNAMENT OF THE TOWNS

O-Level Paper

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- 1 [3] 2003 dollars are placed into N purses, and the purses are placed into M pockets. It is known that N is greater than the number of dollars in any pocket. Is it (always) true that there is a purse with less than M dollars in it?
- 2 [3] 100-gon made of 100 sticks. Could it happen that it is not possible to construct a polygon from any lesser number of these sticks?
- 3 [4] Point M is chosen in $\triangle ABC$ so that the radii of the circumcircles of $\triangle AMC$, $\triangle BMC$, and $\triangle BMA$ are no smaller than the radius of the circumcircle of $\triangle ABC$. Prove that all four radii are equal.
- 4 [5] In the sequence 00, 01, 02, 03, . . . , 99 the terms are rearranged so that each term is obtained from the previous one by increasing or decreasing one of its digits by 1 (for example, 29 can be followed by 19, 39, or 28, but not by 30 or 20). What is the maximal number of terms that could remain on their places?
- 5 [5] Prove that one can cut $a \times b$ rectangle, $\frac{b}{2} < a < b$, into three pieces and rearrange them into a square (without overlaps and holes).

Keep the problem set.

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