Juniors

(Grades up to 10)

International Mathematics
TOURNAMENT OF THE TOWNS

A-Level Paper **Spring 2003.**

1 [**4**] Johnny writes down quadratic equation

$$ax^2 + bx + c = 0$$

with positive integer coefficients a, b, c. Then Pete changes one, two, or none "+" signs to "−". Johnny wins, if both roots of the (changed) equation are integers. Otherwise (if there are no real roots or at least one of them is not an integer), Pete wins.

Can Johnny choose the coefficients in such a way that he will always win?

2 [**4**] $\triangle ABC$ is given. Prove that $R/r > a/h$, where R is the radius of the circumscribed circle, r is the radius of the inscribed circle, a is the length of the longest side, h is the length of the shortest altitude.

3 In a tournament, each of 15 teams played with each other exactly once. Let us call the game *"odd"* if the total number of games previously played by both competing teams was odd.

(a) [**4**] Prove that there was at least one "odd" game.

(b) [**3**] Could it happen that there was exactly one "odd" game?

4 [**7**] A chocolate bar in the shape of an equilateral triangle with side of the length n, consists of triangular chips with sides of the length 1, parallel to sides of the bar. Two players take turns eating up the chocolate.

Each player breaks off a triangular piece (along one of the lines), eats it up and passes leftovers to the other player (as long as bar contains more than one chip, the player is not allowed to eat it completely).

A player who has no move or leaves exactly one chip to the opponent, loses.

For each n, find who has a winning strategy.

5 [**7**] What is the largest number of squares on 9×9 square board that can be cut along their both diagonals so that the board does not fall apart into several pieces?

6 [**7**] A trapezoid with bases AD and BC is circumscribed about a circle, E is the intersection point of the diagonals. Prove that $\angle AED$ is not acute.