

International Mathematics
TOURNAMENT OF THE TOWNS

A-Level Paper

Fall 2003.

- 1 [4] An increasing arithmetic progression consists of one hundred positive integers. Is it possible that every two of them are relatively prime?
- 2 [5] Smallville is populated by unmarried men and women, some of them are acquainted. Two city's matchmakers are aware of all acquaintances. Once, one of matchmakers claimed: "I could arrange that every brunette man would marry a woman he was acquainted with". The other matchmaker claimed "I could arrange that every blonde woman would marry a man she was acquainted with". An amateur mathematician overheard their conversation and said "Then both arrangements could be done at the same time! " Is he right?
- 3 [5] Find all positive integers k such that there exist two positive integers m and n satisfying $m(m + k) = n(n + 1)$.
- 4 [6] Several squares on a 15×15 chessboard are marked so that a bishop placed on any square of the board attacks at least two of marked squares. Find the minimal number of marked squares.
- 5 [7] A point O lies inside of the square $ABCD$. Prove that the difference between the sum of angles OAB, OBC, OCD, ODA and 180° does not exceed 45° .
- 6 [7] An ant crawls on the outer surface of the box in a shape of rectangular parallelepiped. From ant's point of view, the distance between two points on a surface is defined by the length of the shortest path ant need to crawl to reach one point from the other. Is it true that if ant is at vertex then from ant's point of view the opposite vertex be the most distant point on the surface?
- 7 [8] Two players in turns play a game. Each player has 1000 cards with numbers written on them; namely, First Player has cards with numbers $2, 4, \dots, 2000$ while Second Player has cards with numbers $1, 3, \dots, 2001$. In each his turn, a player chooses one of his cards and puts it on a table; the opponent sees it and puts his card next to the first one. Player, who put the card with a larger number, scores 1 point. Then both cards are discarded. First Player starts. After 1000 turns the game is over; First Player has used all his cards and Second Player used all but one. What are the maximal scores, that players could guarantee for themselves, no matter how the opponent would play?