

**International Mathematics
TOURNAMENT OF THE TOWNS**

Senior O-Level Paper (Grades 11-OAC)

Spring 2001.

- 1 [4] The last digit of the number $x^2 + xy + y^2$ is zero (where x and y are positive integers). Prove that two last digits of this number are zeroes.
- 2 [5] Triangle ABC and its mirror reflection $A'B'C'$ are arbitrarily placed on the plane. Prove that the midpoints of the segments AA' , BB' and CC' lie on the same straight line.
- 3 [5] There are 6 pieces of cheese all of different weight. For any two of them one can determine, just by looking at them, which of them is the heaviest.

Given that it is possible to divide them into two groups of equal weights (three pieces in each group) demonstrate how to find these groups by performing two weightings on the regular balance.
- 4 [5] In how many ways can one place the numbers from 1 to 100 in a 2×50 -rectangle (divided into 100 little squares) so that any two consecutive numbers are always placed in squares with a common side?
- 5 [6] Does there exist a regular triangular prism that can be covered (without overlapping) by different equilateral triangles? (One is allowed to bend the triangles around the edges of the prism.)