International Mathematics
TOURNAMENT OF THE TOWNS

Junior A-Level Paper **Spring 2002.**

1. Let a, b and c be the sides of a triangle. Prove that $a^3 + b^3 + 3abc > c^3$.

2. A game is played on a 23×23 board. The first player controls two white chips which start in the bottom-left and the top-right corners. The second player controls two black ones which start in the bottom-right and the top-left corners. The players move alternately. In each move, a player can move one of the chips under control to a vacant square which shares a common side with its current location. The first player wins if the two white chips are located on two squares sharing a common side. Can the second player prevents the first player from winning?

3. Let E and F be the respective midpoints of sides BC and CD of a convex quadrilateral $ABCD$. Segments AE, AF and EF cut $ABCD$ into four triangles whose areas are four consecutive positive integers. Determine the maximal area of triangle BAD.

4. There are n lamps in a row, some of which are on. Every minute, all the lamps already on will go off. Those which were off and were adjacent to exactly one lamp that was on will go on. For which n can one find an initial configuration of which lamps are on, such that at least one lamp will be on at any time?

5. An acute triangle was dissected by a straight cut into two pieces which are not necessarily triangles. Then one of the pieces was dissected by a straight cut into two pieces, and so on. After a few dissections, it turned out that all the pieces are triangles. Can all of them be obtuse?

6. In an increasing infinite sequence of positive integers, every term starting from the 2002-th term divides the sum of all preceding terms. Prove that every term starting from some term is equal to the sum of all preceding terms.

7. Some domino pieces are placed in a chain according to the standard rules. In each move, we may remove a sub-chain with equal numbers at its ends, turn the whole sub-chain around, and put it back in the same place. Prove that for every two legal chains formed from the same pieces and having the same numbers at their ends, we can transform one to the other in a finnite sequence of moves.

Note: The problems are worth 4, 4, 6, 7, 7, 7 and 8 points respectively.