Juniors
(Grades up to 10)

International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper **Fall 2002.**

1 [**4**] In a convex 2002-gon several diagonals are drawn so that they do not intersect inside of the polygon. As a result, the polygon splits into 2000 triangles.

Is it possible that exactly 1000 triangles have diagonals for all of their three sides?

2 [**5**] Each of two children (John and Mary) selected a natural number and communicated it to Bill. Bill wrote down the sum of these numbers on one card and their product on another, hid one card and showed the other to John and Mary.

John looked at the number (which was 2002) and declared that he was not able to determine the number chosen by Mary. Knowing this, Mary said that she was also not able to determine the number chosen by John.

What was the number chosen by Mary?

3

a) [**1**] A test was conducted in a class. It is known that at least $\frac{2}{3}$ of the problems were hard: each such problem was not solved by at least $\frac{2}{3}$ of the students. It is also known that at least $\frac{2}{3}$ of students passed the test: each such student solved at least $\frac{2}{3}$ of the suggested problems. *Is this situation possible?*

b) [**2**] The same question with $\frac{2}{3}$ replaced by $\frac{3}{4}$.

c) [**2**] The same question with $\frac{2}{3}$ replaced by $\frac{7}{10}$.

4) [**5**] 2002 cards with the numbers 1, 2, 3,...,2002 written on them are put on a table face up. Two players in turns pick up a card from the table until all cards are gone. The player who gets the last digit of the sum of all numbers on his cards larger than his opponent, wins.

Who has a winning strategy and how one should play to win?

5) [**5**] An angle and a point A inside of it are given. Is it possible to draw through A three straight lines so that on either side of the angle one of three points of intersection of these lines be the midpoint between two other points of intersection with that side?

Keep the problem set.
Visit: `http://www.math.toronto.edu/oz/turgor/`