

Juniors

(Grades up to 10)

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A-Level Paper

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- 1 [4] There are 2002 employees in a bank. All the employees came to celebrate the bank's jubilee and were seated around one round table. It is known that the difference in salaries of any two employees sitting next to each other is 2 or 3 dollars. Find the maximal difference in salaries of two employees, if it is known that all the salaries are different.
- 2 [5] All the species of plants existing in Russia are catalogued (numbered by integers from 2 to 20 000; one after another, without omissions or repetitions). For any pair of species, the greatest common divisor of their catalogue numbers was calculated and recorded, but the catalogue numbers themselves were lost (computer error). Is it possible to restore the catalogue number for each species from that data?
- 3 [6] The vertices of a 50-gon divide a circumference into 50 arcs, whose lengths are 1, 2, 3, ..., 50, in some order. It is known that lengths of any pair of "opposite" arcs (corresponding to opposite sides of the polygon) differ by 25. Prove that the polygon has two parallel sides.
- 4 [6] Point P is chosen in triangle ABC so that $\angle ABP$ is congruent to $\angle ACP$, while $\angle CBP$ is congruent to $\angle CAP$. Prove that P is the intersection point of the altitudes of the triangle.
- 5 [7] A convex N -gon is divided by diagonals into triangles so that no two diagonals intersect inside of the polygon. The triangles are painted in black and white so that any two triangles with common side are painted in different colors. For each N , find the maximal difference between the numbers of black and white triangles.
- 6 [9] There is a large pile of cards. On each card one of the numbers $1, 2, \dots, n$ is written. It is known that the sum of all numbers of all the cards is equal to $k \cdot n!$ for some integer k . Prove that it is possible to arrange cards into k stacks so that the sum of numbers written on the cards in each stack is equal to $n!$.
- 7 a) [5] A power grid has the shape of a 3×3 lattice with 16 nodes (vertices of the lattice) joined by wires (along the sides of the squares). It may have happened that some of the wires are burned out. In one test technician can choose any pair of nodes and check if electrical current circulates between them (that is, check if there is a chain of intact wires joining the chosen nodes). Technician knows that current will circulate from any node to any other node. What is the least number of tests which is required to demonstrate this?
- 7 b) [5] The same question for a grid in the shape of a 5×5 lattice (36 nodes).

Keep the problem set.

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