PROBLEMS OF TOURNAMENT OF TOWNS

Spring 2001, Level 0, Senior (grades 11-OAC)

Problem 1 [3] A bus that moves along a 100 km route is equipped with a computer, which predicts how much more time is needed to arrive at its final destination. This prediction is made on the assumption that the average speed of the bus in the remaining part of the route is the same as that in the part already covered. Forty minutes after the departure of the bus, the computer predicts that the remaining travelling time will be 1 hour. And this predicted time remains the same for the next 5 hours. Could this possibly occur? If so, how many kilometers did the bus cover when these 5 hours passed? (Average speed is the number of kilometers covered divided by the time it took to cover them.)

Problem 2 [4] The decimal expression of the natural number a consists of n digits, while that of a^3 consists of m digits. Can $n + m$ be equal to 2001?

Problem 3 [4] Points X and Y are chosen on the sides AB and BC of the triangle $\triangle ABC$. The segments AY and CX intersect at the point Z. Given that $AY = YC$ and $AB = ZC$ prove that the points B, X, Z, and Y lie on the same circle.

Problem 4 [5] Two persons play a game on a board divided into 3×100 squares. They move in turn: the first places tiles of size 1×2 lengthwise (along the long axis of the board), the second, in the perpendicular direction. The loser is the one who cannot make a move. Which of the players can always win (no matter how his opponent plays), and what is the winning strategy?

Problem 5 [5] Nine points are drawn on the surface of a regular tetrahedron with an edge of 1 cm. Prove that among these points there are two located at a distance (in space) no greater than 0.5 cm.