International Mathematics

22nd Tournament of Towns

Ordinary Level

February 25, 2001

Junior (Grades 7, 8, 9 and 10)

Your total score is based on the three problems for which you earn the most points; the scores for the individual parts of a single problem are added. Points for each problem are shown in brackets ( ).

1. (3) The natural number n can be replaced by ab if $a + b = n$, where a and b are natural numbers. Can the number 2001 be obtained from 22 after a sequence of such replacements?

2. (4) One of the midlines of a triangle is longer than one of its medians. Prove that the triangle has an obtuse angle.

3. (4) Twenty kilograms of cheese are on sale in a grocery store. Several customers are lined up to buy this cheese. After a while, having sold the demanded portion of cheese to the next customer, the salesgirl calculates the average weight of the portions of cheese already sold and declares the number of customers for whom there is exactly enough cheese if each customer will buy a portion of cheese of weight exactly equal to the average weight of the previous purchases. Could it happen that the salesgirl can declare, after each of the first 10 customers has made their purchase, that there just enough cheese for the next 10 customers? If so, how much cheese will be left in the store after the first 10 customers have made their purchases? (The average weight of a series of purchases is the total weight of the cheese sold divided by the number of purchases.)

4. a. (2) There are 5 identical paper triangles on the table. Each can be moved in any direction parallel to itself (i.e., without rotating it). Is it true that then any one of them can be covered by the 4 others?

   b. (3) There are 5 identical equilateral paper triangles on the table. Each can be moved in any direction parallel to itself. Prove that any one of them can be covered by the 4 others in this way.

5. (5) On a square board divided into 15×15 little squares there are 15 rooks that do not attack each other. Then each rook makes one move like that of a knight. Prove that after this is done a pair of rooks will necessarily attack each other.