International Mathematics
TOURNAMENT OF THE TOWNS

Senior O-Level Paper **Fall, 2001.**

1. An altitude of a pentagon is the perpendicular drop from a vertex to the opposite side. A median of a pentagon is the line joining a vertex to the midpoint of the opposite side. If the five altitudes and the five medians all have the same length, prove that the pentagon is regular.

2. There exists a block of 1000 consecutive positive integers containing no prime numbers, namely, $1001! + 2$, $1001! + 3$, ..., $1001! + 1001$. Does there exist a block of 1000 consecutive positive integers containing exactly five prime numbers?

3. On an east-west shipping lane are ten ships sailing individually. The first five from the west are sailing eastwards while the other five ships are sailing westwards. They sail at the same constant speed at all times. Whenever two ships meets, each turns around and sails in the opposite direction. When all ships have returned to port, how many meetings of two ships have taken place?

4. On top of a thin square cake are triangular chocolate chips which are mutually disjoint. Is it possible to cut the cake into convex polygonal pieces each containing exactly one chip?

5. The only pieces on an 8×8 chessboard are three rooks. Each moves along a row or a column without running to or jumping over another rook. The white rook starts at the bottom left corner, the black rook starts in the square directly above the white rook and the red rook start in the square directly to the right of the white rook. The white rook is to finish at the top right corner, the black rook in the square directly to the left of the white rook and the red rook in the square directly below the white rook. At all times, each rook must be either in the same row or the same column as another rook. Is it possible to get the rooks to their destinations?

Note: Each problem is worth 4 points.