

How To Solve. VI.

Strategy Games.

- A *Strategy Game* is a contest between *two* players that results in a *win* one of the players (thus a *loss* of the opponent) or in a *tie* if a game admits it.
- Players *alternate* their moves.
- At any moment a game is defined by *position* that could be changed *only* by player's move.
- Some positions are declared as *winning* positions while the others as *losing* positions (unless it is a game with a tie).
- The goal of each player is to reach a winning position (in other words, to force his opponent to a losing position).
- Some strategy games admit a tie. That means that neither of players can reach a winning position.
- Examples of strategy games: chess, checkers, tic-tac-toe, etc.
- Domino, games with dice, most of the card games are not strategy games (position is defined not only by move of a player, but also depends on random factors, such as distribution of cards, etc).
- *Winning strategy* is an algorithm that leads one player to a win, while *tie strategy* is an algorithm that allows one player either to win or to draw.

Hints for Strategy Searching

- Correspondence –
Sometimes one can find reciprocal move that lead to win (provided by symmetry or by pairing or by complement of a set).
- Change one's position –
If one can use the opponent's strategy that would mean that one situation is worse than situation of the opponent. For example, win or draw is provided if one can reach some certain position according to one's will (or to force the opponent to some certain position).
- Backwards search –
Starting from the end, in succession of descents are defined the winning and losing positions for each player. At some moment position is declared a winning if there is a move from it to a position previously defined as a losing position. On the other hand, position is declared a losing if any move from it leads to a position previously defined as a winning position.

Problems

1. Two players A and B in turns place coins (cents) on an empty place of a circle table. The player who can not place a coin (not enough place) loses. Which of the players has a winning strategy, the first or the second?
2. Two girls, Anna and Betty in turns tear off petals of cauliflower. On each turn a girl can tear off one or two neighboring petals. The girl who tears off the last petal is lucky. Which of the girls can provide a luck for herself ? What is the strategy that guarantees a luck?
3. Consider the game “Tic-Tac-Toe” on an infinite board. Does the second player have a winning strategy?
4. Consider the chess game with double move (each player moves a figure twice). Prove that Blacks have no winning strategy.
5. White and black checkers occupy the leftmost and rightmost cells of 1×20 board respectively. Two players in turns play the following game: each of the players moves its checker one or two steps in either direction. It is not allowed to jump over the other checker nor to land on a square which is already occupied. The player who cannot make a move loses. Which of the players has a winning strategy? Player that moves the white checker starts first.
6. Two players in turns colour the sides of n -gon. The first player colours any side that has 0 or 2 common vertices with already coloured sides. The second player colours any side that has exactly 1 common vertex with already coloured sides. The player who cannot move loses. For which n the second player has a winning strategy?
7. There are 25 nuts in a pile. Two players in turns pick up two, three or five nuts at the time. The player who takes the last nut, wins. Which of the players has a winning strategy?
8. There are 2 piles of candies, 18 and 20. Two children, Anna and Boris in turns pick up any number of candies from one of the piles. The child who takes the last candy wins. Anna starts first. Which of the children has a winning strategy?
9. There are 2 piles of candies, 18 and 23. Two children, Anna and Boris in turns eat up one of the piles, dividing the other into 2 piles. Child who cannot divide a pile (if there is only 1 candy left) loses. Boris starts first. Which of the children has a winning strategy?
10. There are two piles of candies, 9 candies each. Two players in turn play the following game. On each turn one player moves one candy from one of the piles to another pile and then eats 2 candies from any pile. The player who cannot move loses. Which of the players has the winning strategy, the first or the second?