# 1 Math Club November 26, 2000

## Warm-Up Problems

**Problem 1.** There are two ropes. It is known that each of them burns out completely in exactly 1 hour. However, the pace of burning is not uniform. How one can measure 45 minutes interval, using these ropes and a box of matches?

**Problem 2.** Everybody in Wonderland is either a Liar or a Truthteller. A visitor overheard a conversation between four natives.

$B$ to $A$: Your are a Liar!
$C$ to $B$: It is you, who is a Liar!
$D$ to $C$: Both of them are Liars!
$D$ to $C$ (again): And you are a Liar as well...
Could you tell who is who?

## Olympiads archive

**Problem 1** (Russian Olympiad). The Council of Wizards is tested in the following way: The King lines the wizards up in a line and places on the head of each of them either a white hat or a blue hat or a red hat. Each wizard sees the colors of hats of the people standing in front of him, but he neither sees the color of his hat nor the colors of hats of the people standing behind. Every minute some of the wizards must announce one of the three colors (it is allowed to speak out just once). After completion of this procedure the King executes all the wizards who failed to guess the right color of their hats. Prior to this ceremony all 100 members have agreed to minimize the number of executions. How many of them are definitely secure against the punishment?

**Problem 2**. (a) A magician draws 5 cards from a 52-card deck at random, looks at them, and arranges them in a row from left to right, with one card, not necessarily the first. facing down and the others facing up. The assistant "guesses" the card which is face down. Prove that the performers can agree on a system which makes this work every time.

(b) The second trick is the same except that the magician arranges only four cards, facing up, but keeps the fifth card hidden. Can the performers still agree on a system which enables the assistant to "guess," the hidden card?

**How to solve problems?**

**Looking for a relative problem**

If a problem is hard, try to find and solve simpler problem, related to it. This often gives you a clue to the original problem. These advices may be helpful:

- Consider a special (simpler) case and generalize the idea of it's solution afterwards.
- Break the problem down into the smaller ones.
- Generalize the problem. For example, you can replace a given number by a variable.
- Reduce the problem to the simpler one.

**Problem 1.** We put "+" in one corner cell of $5 \times 5$-table and "−" in all other cells. One can flip all the signs in any column or a row. Can one get in a finite number of steps all pluses?

**Problem 2.** One can easily cut $3 \times 3 \times 3$-cube into 27 smaller cubes by 6 cuts. Can one reduce the number of cuts if one is allowed to move pieces between cuts?

**Problem 3.** Solve equation $(x^2 + x - 3)^2 + 2x^2 + 2x - 5 = 0$.

**Problem 4.** Compare two fractions: $\frac{31415927}{2345678}$ **?** $\frac{31415928}{2345679}$

**Problem 5.** Prove that the sum of interior angles a convex polygon equals $180^\circ \cdot (n-2)$.

**Problem 6.** Prove that $n(n+1)(n+2)$ is divisible by 6 for each natural $n$.

**Problem 7.** Construct a tangent to two given circles. Hint. Replace one circle by a point.