

How To Solve. VII.

Estimate-Example problems

In an *estimate-example problems*, one is asked to find the smallest (largest) value of M that satisfies a given property. The problem is tackled in two steps:

- first, one must establish an *estimate* for M and
- second, find an *example* where M is reached. Both parts are equally important; however, they may greatly vary in difficulty.

Problems

1. There are 15 pikes in the pond. Pikes can eat one another. Pike is sated if it ate 3 other pikes (sated or not). What is the maximal number of pikes that could be sated? (Sated pike eaten later also counts).
2. What is the maximal number of
 - (a) Rooks
 - (b) Bishops
 - (c) Knightsone can place on 8x8 chessboard so that no figure is under attack?
3. Find all values of $n > 2$ (positive integer) for which it is possible to arrange all integers from 1 to n on a circle so that the sum of any two neighboring numbers is divisible by the number next to them in clockwise direction.
4. An ant crawls on the surface of a cube going from vertex to vertex either by edge or by face diagonal. It is not allowed to intersect the path or to visit the same vertex twice. Find the maximal length of the path from one vertex to the opposite vertex of the cube.
5. Each term of a sequence of natural numbers is obtained from the previous term by adding to it its largest digit. What is the maximal number of successive odd terms in such a sequence?
(Tournament of the Towns 2003, O-Level, Spring Round)
6. 25 checkers are placed on 25 leftmost squares of $1 \times N$ board. A checker can either move to the empty adjacent square to its right or jump over an adjacent right checker to the next square if it empty. moves to the left are not allowed. Find minimal N such that all the checkers could be placed in the row of 25 squares but in the reverse order.
(Tournament of the Towns 2003, O-Level, Fall Round)
7. A chess piece moves as follows: it can jump 8 or 9 squares either vertically or horizontally. It is not allowed to visit the same square twice. At most, how many squares can this piece visit on a 15×15 board (it can start from any square)?
(Tournament of the Towns 2005, O-Level, Fall Round)

8. There are eight identical Black Queens in the first row of a chessboard and eight identical White Queens in the last row. The Queens move one at the time, horizontally, vertically or diagonally by any number of squares as long as no other Queens are in the way. Black and White Queens move alternately. What is the minimal number of moves required for interchanging the Black and White Queens?

(Tournament of the Towns 2005, O-Level, Spring Round)