

Coloring

The *coloring* can be used as idea of solution. The coloring can be either predefined (for example, the coloring of a chessboard) or it can be created (with some specific properties).

Problems

1. Two opposite angles are cut out of a chessboard. Is it possible to pave this board with dominos (each domino occupies two squares)?
2. Is it possible for Knight to visit each square of 9×9 board exactly once and to return to the initial square?
3. 25 ants are sitting on a 5×5 board, one in each square. At the same time each ant moves to adjacent square (by side). Prove that at least one square of the board will be empty.
4. Is it possible to pave a 10×10 board by 1×4 stripes?
5. Is it possible to pave a 8×8 board by 1×4 stripes and one “L” shaped figure, consisting of four squares?
6. Is it possible to construct a $6 \times 6 \times 6$ cube using $1 \times 2 \times 4$ blocks?
7. Is it possible to construct a $3 \times 3 \times 3$ cube with a $1 \times 1 \times 1$ hole in the center using $1 \times 1 \times 2$ blocks?
8. A plane is colored with two colors. Prove that there exist two points of the same color that are exactly 1 cm apart.
9. A plane is colored with three colors. Prove that there exist two points of the same color that are exactly 1 cm apart.