Some Formulae

 # 1. Trig. formula:
$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b.$
$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b.$

 # 2. Laplacian in 3-d: $\Delta = \nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

 # 3. Divergence in 3-d: $\text{div}\vec{F} = \nabla \cdot \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$.

 # 4. Laplacian in the cylindrical coordinates: With $x = r\cos\theta$, $y = r\sin\theta$, and $z = z$, we have

$$\Delta u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2}$$

 # 5. Laplacian in the spherical coordinates: With $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, and $z = \rho\cos\phi$, we have

$$\Delta u = \frac{1}{\rho^2}\frac{\partial}{\partial \rho}\left(\rho^2\frac{\partial u}{\partial \rho}\right) + \frac{1}{\rho^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial u}{\partial \phi}\right) + \frac{1}{\rho^2\sin^2\phi}\frac{\partial^2 u}{\partial \theta^2}$$

 # 6. Divergence Theorem:

$$\iiint_\Omega \text{div}\vec{F}dV = \iint_{\partial\Omega} \vec{F}\cdot\widehat{n}dS,$$

where $\text{div}\vec{F} = \nabla \cdot \vec{F}$.

 # 7. Fourier Series: Given f on $[-L, L]$, we have

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L},$$

where $a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$,

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx, \quad \text{and} \quad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx.$$

 # 8. Example of a boundary value problem on $[-L, L]$:

$$\begin{cases} \frac{d^2F(x)}{dx^2} = -\lambda F(x) \\ F(-L) = F(L) \\ \frac{dF}{dx}(-L) = \frac{dF}{dx}(L). \end{cases} \tag{1}$$

Then, the eigenvalues λ are given by $\lambda = \left(\frac{n\pi}{L}\right)^2$, and the corresponding eigenfunctions are $\cos\frac{n\pi x}{L}$ and $\sin\frac{n\pi x}{L}$.