Some Formulae

Assume that all the functions are "nice".

 # 1. Fourier transform:

$\widehat{f}(\omega) =$ " Fourier transform of $f(x)$" $= \frac{1}{2\pi}\int_{\mathbb{R}} f(x)e^{i\omega x}dx$.

$g^{\vee}(x) =$ "inverse Fourier transform of $g(\omega)$".

 # 2. Inverse Fourier transform of the Gaussian:

$$(e^{-\beta\omega^2})^{\vee}(x) = \sqrt{\frac{\pi}{\beta}}e^{-\frac{x^2}{4\beta}}.$$

 # 3. Fundamental solution $\Phi_k(x,t)$ for the heat equation $u_t = ku_{xx}$:

$$\Phi_k(x,t) = \frac{1}{\sqrt{4\pi kt}}e^{\frac{-x^2}{4kt}}.$$

i.e. $u(x,t) = \big(\Phi_k(\cdot,t) * f\big)(x)$ solves $u_t = ku_{xx}$ with $u(x,0) = f(x)$.

 # 4. Green's function for Poisson's equation:

$$G(\vec{x},\vec{x}_0) = \frac{1}{2\pi}\ln|\vec{x}-\vec{x}_0| \text{ on } \mathbb{R}^2, \quad G(\vec{x},\vec{x}_0) = -\frac{1}{4\pi|\vec{x}-\vec{x}_0|} \text{ on } \mathbb{R}^3.$$

 # 5. Poisson formula for the disk of radius a:

$$u(r,\theta) = \frac{1}{2\pi}\int_0^{2\pi} h(\theta_0)\frac{a^2-r^2}{r^2+a^2-2ar\cos(\theta-\theta_0)}d\theta_0$$

 # 6. Laplacian in 3-d: $\Delta = \nabla^2 = \nabla\cdot\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

 # 7. Divergence in 3-d: $\text{div}\vec{F} = \nabla\cdot\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$.

 # 8. Laplacian in the cylindrical coordinates: With $x = r\cos\theta$, $y = r\sin\theta$, and $z = z$, we have

$$\Delta u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2} + \frac{\partial^2 u}{\partial z^2}$$

 # 9. Divergence Theorem:

$$\iiint_{\Omega} \text{div}\vec{F}dV = \iint_{\partial\Omega} \vec{F}\cdot\widehat{n}dS,$$

where $\text{div}\vec{F} = \nabla\cdot\vec{F}$.

 # 10. Fourier Series: Given f on $[-L,L]$, we have

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L},$$

where $a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$,

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx, \quad \text{and} \quad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx.$$